9.5

October 20, 2000 | MATHEMATICS 110 (31) Quiz #2 | Total Marks - 25 Time: 45 minutes

Last Name: ____________ Student Number: ____________

[8] 1. Evaluate the following limits (SHOW ALL YOUR WORK):

a) $\lim_{x\to\infty} \frac{2x^3 - 5x + 1}{5x - 3x^3} =$ $\frac{\frac{2x^3}{x^3} - \frac{5x}{x^3} + \frac{1}{x^3}}{\frac{5x}{x^3} - \frac{3x^3}{x^3}} = \frac{2}{-3} = \frac{-2}{3}$

asymtote at $\frac{-2}{3}$ horizontal

b) $\lim_{x\to-\infty} \frac{\sqrt{x^2-2}}{x-2} =$ $\lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x} - \frac{2}{x}}}{\frac{x}{x} - \frac{2}{x}}$ $-\lim_{x\to-\infty} \frac{\sqrt{\frac{x^2}{x}}}{\frac{x}{x}} = \frac{\sqrt{x}}{1} = \frac{\sqrt{-\infty}}{1}$

$\frac{-\sqrt{2}}{x}$ / $x - \frac{2}{x}$

$\frac{\frac{x}{x}}{\frac{x}{x}} = 1$

$\frac{x^2-2}{x-2} \sqrt{x^2+2}$

asymtote at 1 horizontal

c) $\lim_{x\to 0} \frac{\sin x \cos(x+\pi)}{x} =$ $-1 \le \sin x \le 1$

$-1 \le \sin x \cos(x+\pi) \le 1$

$\frac{-1}{x} \le \frac{\sin x \cos(x+\pi)}{x} \le \frac{1}{x}$ $= 0$

d) $\lim_{x\to 0} \frac{e^{1+x} - e}{x} =$

$= 1$

[3] 2. Find all vertical and horizontal asymptotes of

$$f(x) = \frac{1-|x|}{1+x}.$$

Justify your answer by evaluating all the relevant limits.

$\frac{1-|x|}{}$ $\frac{\frac{1}{x} - \frac{|x|}{x}}{}$ -2 $\lim \frac{1-|x|}{1+x} = \frac{1}{1} = \frac{2}{2}$

[9] **3. Differentiate the following functions (SHOW ALL YOUR WORK):**

a) $f(x) = 3x^5 - 6x + 2$

$\frac{dy}{dx} = \frac{d3x^5}{dx} - \frac{d6x}{dy} + \frac{d2}{dx}$

$f'(x) = 15x^4 - 6$

b) $f(x) = \dfrac{3\sin x - 2}{\cos x + x^{10}}$

$f'g + fg' = \frac{d3}{dx} \cdot \sin x - 2 + 3\left(\frac{d \sin x - 2}{dx}\right)$

$\frac{f'g - fg'}{g^2}$

$(3\cos x - 2)(\cos x + x^{10}) - (3\sin x - 2)(-\sin x + x^{10}$

$= 0 + 3(\cos x - 2 \cdot 1)$

$f' = 3\cos x - 2$ } sub in to

$g' = -\sin x + x^{10} \cdot 1 + 10x^9$

$f'(x)$

$= \dfrac{(3\cos x - 2)(\cos x + x^{10}) - (3\sin x - 2)(-\sin x + x^{10})(1 + 10x^9)}{(\cos x + x^{10})^2}$

-1.5

c) $f(x) = xe^x \cos x - 10\sqrt{x} + \pi$

-3

$\frac{dy}{dx} = \frac{dxe^x}{dx} \cdot \cos x - 10\sqrt{x} - \pi + \frac{dy \cos x - 10\sqrt{x} + \pi}{dx} \cdot xe^x$

$= (x^2 e^x)(\cos x - 10\sqrt{x} - \pi) + (-\sin x - 10\sqrt{x} + \pi)(\frac{dx - 10}{dx} \cdot \sqrt{x} + \pi + \frac{d\sqrt{x} + \pi}{dx} \cdot x - 10)(xe$

$= (xe^x)(\cos x - 10\sqrt{x} + \pi) + (-\sin x - 10\sqrt{x} + \pi)(1 \cdot \sqrt{x} + \pi + \frac{1}{2}x^{-1/2} \cdot x - 10)(xe^x$

$= (xe^x)(\cos x - 10\sqrt{x} + \pi) + (-\sin x - 10\sqrt{x} + \pi)(\sqrt{x} + \pi + \frac{1}{2}x^{-1/2})(x - 10)(xe$

[2] 4. The next two questions have to do with the definition of the derivative.

a) State the definition of $f'(a)$ for a point $x = a$ in the domain of the function $f(x)$.

$$\lim_{h \to 0} = \frac{f(a+h) - f(a)}{h}$$

b) State whether the following statement is TRUE or FALSE and justify your answer. "Every function $f(x)$ which is continuous at $x = a$ is also differentiable at $x = a$."

True

If it is Condinuous than that means their is a line for a graph and that Can be differentiable.

[3] 5. Find an equation for the tangent line to the curve $y = \sin x - x$ at the point on the curve with $x = \pi/2$.

$y' = \cos x - x \cdot 1 - 1$

$y' = \cos x$

$y = \sin\frac{\pi}{2} - \frac{\pi}{2}$

$= \sin 0$

$y = 0$

$x = \frac{\pi}{2}$

$m = \cos x$

$\frac{\cos x}{1} = \frac{y - 0}{x - \frac{\pi}{2}}$

NAME: [redacted] I.D.: [redacted]

MATH 110 – TEST # 2 – 2001 – November 1, 2001
Time: 75 minutes. No Calculators, Closed Book.

PART 1: *Choose your answer from the options (A), (B), $\cdots$, (H) and write the letter in the space provided. Each correct answer has a value of 4%, an incorrect answer has a value of 0%.*

1. If $f(x) = 6x^{1/3} - \frac{1}{3}x^6$, then $f'(x) =$

(A) $2x^{-1/2} - 2x^5$ (B) $2x^{-2/3} - 2x^5$ (C) $2x^{-1/3} - 2x^2$

(D) $2x^{-2/3} - 2x^7$ (E) $18x^{4/3} - 2x^7$ (F) $2x^{1/2} - 2x^2$

(G) $6x^{-2/3} - \frac{1}{3}x^5$ (H) $\dfrac{2(1 - x^{13/3})}{x^{2/3}}$

Answer: B

2. If $g(t) = t^2 \tan t$, then $g'(t) =$

(A) $2t \tan t$ (B) $t^2 \sec^2 t$ (C) $t^2 \tan t \sec t$

(D) $2t + \sec^2 t$ (E) $2t \tan t + t^2 \sec^2 t$ (F) $2t \sec^2 t$

(G) $2t \tan t + t^2 \sec t \tan t$ (H) $2t \sec t \tan t$

Answer: E

3. If $y = \dfrac{x^3}{1 + x^4}$, then $\dfrac{dy}{dx} =$

(A) $\dfrac{3x^2 + x^6}{(1 + x^4)^2}$ (B) $\dfrac{x^6 + 3x^2}{(1 + x^4)^2}$ (C) $\dfrac{7x^6 - 3x^2}{(1 + x^4)^2}$

(D) $\dfrac{3x^2 - 7x^6}{(1 + x^4)^2}$ (E) $\dfrac{3x^2 - x^6}{(1 + x^4)^2}$ (F) $\dfrac{3x^2 - x^6}{1 + x^4}$

(G) $\dfrac{x^6 - 3x^2}{1 + x^4}$ (H) $\dfrac{4x^3 - x^6}{(1 + x^4)^2}$

Answer: E

4. If $u = \cos(\ln t)$, then $u'(t) =$

(A) $\cos\left(\frac{1}{t}\right)$ (B) $-\sin(\ln t)$ (C) $-\sin\left(\frac{1}{t}\right)$

(D) $1/\cos(\ln t)$ (E) $\frac{-\sin t}{t}$ (F) $\frac{-\sin(\ln t)}{t}$

(G) $-\sin(e^t)$ (H) $\sin\left(\frac{1}{t}\right)$

Answer: F

5. If $f(x) = \dfrac{e^{2x}}{1+\sin x}$, then (read carefully) $f'(0) =$

(A) $\dfrac{(1+\sin x)2e^{2x}) - e^{2x}\cos x}{(1+\sin x)^2}$ (B) 1   (C) 2   (D) -1

(E) $\dfrac{-2e^{2x}}{(1+\sin x)^2}$ (F) $\dfrac{2e^{2x}}{\cos x}$ (G) $\dfrac{e^{2x}\cos x - (1+\sin x)2e^{2x}}{(1+\sin x)^2}$ (H) $\frac{1}{2}$

Answer: D

6. If $g(\theta) = \sin(\cos(4\theta))$, then $g'\left(\frac{\pi}{8}\right) =$

(A) $\cos 1$ (B) $-\cos 1$ (C) $4\cos 1$ (D) $-4\sin 1$

(E) $-4\cos 1$ (F) $-4\cos 1 \sin 1$ (G) 4   (H) -4

Answer: H

7. If $h(x) = x^2\sqrt{1+x^2}$, then $h'(2\sqrt{2}) =$

(A) $12\sqrt{2}$ (B) $\frac{16\sqrt{2}}{3}$ (C) $6\sqrt{2}$ (D) $\frac{8\sqrt{2}}{3}$

(E) $\frac{13\sqrt{2}}{3}$ (F) $\frac{52\sqrt{2}}{3}$ (G) $18\sqrt{2}$ (H) 24

Answer: B

8. If $y = 10^x$, then $y' =$

(A) $x\,10^{x-1}$ (B) 10^x (C) $10^x \ln x$ (D) $10^x / \ln 10$

(E) $10^x \ln 10$ (F) $10e^{10x}$ (G) $x\,10^{x-1} \ln 10$ (H) $e\,10^x$

Answer: E

9. If $w = \arcsin(e^t)$, then $w' =$

(A) $\dfrac{1}{1+e^{2t}}$ (B) $\dfrac{e^t}{1+e^{2t}}$ (C) $\dfrac{e^t}{\sqrt{1+e^{2t}}}$ (D) $\dfrac{e^t}{\sqrt{1-e^{2t}}}$

(E) $\sin(e^t)$ (F) $e^t \sin(e^t)$ (G) $\dfrac{e^t}{\sqrt{e^{2t}-1}}$ (H) $e^t \arccos(e^t)$

Answer: C

10. If $A = \arctan(\sqrt{x}\,)$, then $\dfrac{d}{dx}A =$

(A) $\dfrac{1}{1+x}$ (B) $\dfrac{1}{\sqrt{1-x}}$ (C) $\dfrac{\sqrt{x}}{1+x}$ (D) $\dfrac{1}{2\sqrt{x}(1+x)}$

(E) $(x^{1/2} + x^{3/2})^{-1}$ (F) $\text{arcsec}^2(\sqrt{x}\,)$ (G) $\dfrac{1}{2\sqrt{x}}\text{arcsec}^2(\sqrt{x}\,)$ (H) $\text{arccot}(\sqrt{x}\,)$

Answer: D

PART 2: *Provide full solutions showing all work in your answer booklet for the following questions. These questions have a maximum value of 10% each.*

11. For the function $y = \dfrac{2x-1}{x+1}$

(a) find y'

(b) find the points on the graph of y where the slope of the tangent is 1/3.

(c) find the equations of the tangents at these points.

12. Find the equation of the tangent to the graph of $x^4 + y^4 = 17$ at the point $(1, 2)$ on the graph.

13. If $y = 1 + \ln(x^2 + x + 1)$,

    (a) find y'.

    (b) find the equation of the normal to the graph of y at the point on the graph where $x = 0$.

14. If $y = \cos(ax)$ where a is a constant.

    (a) find y' and y''.

    (b) find the values of a for which y satisfies the equation $y'' + 16y = 0$.

15. A particle moves in a vertical line so that its co-ordinate at time t is given by $y = t^3 - 12t, \quad t \geq 0$.

    (a) Find the velocity and acceleration functions.

    (b) When is the particle moving upward and when is it moving downward?

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November 2, 2000 — MATHEMATICS 110 (31) Midterm #2 — Total Marks - 30 Time: 75 minutes

Last Name: ______ Student Number: ______

9.5 [12] 1. Find f' for the following functions.

a) $f(x) = -2(3 - x^8)^{100}$

$$\frac{dy}{dx} = -2\left(\frac{d(3-x^8)^{100}}{dx}\right) + (3-x^8)^{100}\frac{d2}{dx} = 0$$

$$= -2\left(100(3-x^8)^{99}(-8x^7)\right) + (3-x^8)^{100}$$

$$f'(x) = -2\left(100(3-x^8)^{99}\cdot -8x^7\right) + (3-x^8)^{100}$$

$\sin u$ $u = x^2$ $\frac{du}{dx} = 2x$

$u =$ $u^{1/2}$ $\frac{du}{dx}$

b) $f(x) = \left(\dfrac{1-x^3}{4}\right)^{1/2}\sin(x^2)$

$$\frac{d}{dx} = \left(\frac{1-x^3}{4}\right)^{1/2}\frac{d\sin(x^2)}{dx} + \sin(x^2)\, d\left(\frac{1-x^3}{4}\right)^{1/2}$$

$$= \left(\frac{1-x^3}{4}\right)^{1/2}\cos(x^2)\cdot 2x + \sin(x^2)\,\tfrac{1}{2}\left(\frac{1-x^3}{4}\right)^{-1/2}\cdot\frac{1-x^3(0)-(4)(-3x^2)}{4^2}$$

$$f'(x) = \left(\frac{1-x^3}{4}\right)^{1/2}\cdot\left(\cos(x^2)\cdot 2x\right) + \sin(x^2)\cdot\left(\tfrac{1}{2}\left(\frac{1-x^3}{4}\right)^{-1/2}\cdot\frac{+(4)(-3x^2)}{4^2}\right)$$

c) $f(t) = \cos^5\left(\dfrac{\pi}{t}\right) + e^{t^{2/3}} + \ln(\sqrt{t})$ $x = e^{\ln x}$

$$f'(t) = -\sin^5\left(\frac{\pi}{t}\right)\cdot\left(\frac{\pi\cdot\frac{d(t)}{dt} - \cancel{t\frac{d\pi}{dt}}}{t^2}\right) + e^{t^{2/3}} + \frac{1}{\sqrt{t}}$$

$$f'(t) = \left(-\sin^5\left(\frac{\pi}{t}\right)\cdot\left(\frac{\pi}{t^2}\right)\right) + \left(e^{t^{2/3}}\cdot \frac{2}{3}t^{-\frac{1}{3}}\right) + \frac{1}{\sqrt{t}}$$

d) $f(t) = \arctan\left(\dfrac{t+2}{t-1}\right)$ $u = \left(\dfrac{t+2}{t-1}\right)$

$$d(\arctan(u))\cdot du \qquad \frac{du}{dt} = (t+2)\frac{d(t-1)}{dt} + \frac{d(t+2)}{dt}\cdot($$

[4] **2. Find the equation of the tangent to the given curve at the point $(x, y) = (0, 2)$:**

$y^2 - 2xy = 4 - 2x - x^2$

$y^2 = 4 - 2x - x^2 + 2xy$

$\frac{dy}{dx} \cdot \frac{dy^2}{dy} = -2 - 2x + \frac{d2xy}{dx}$

$\frac{dy}{dx} \cdot 2y = -2 - 2x + \frac{d2y}{dy} \cdot \frac{dy}{dx}$

$= -2 - 2x + 2$

$\frac{2y}{2} = \frac{-2x}{2}$?

$y' = -x$

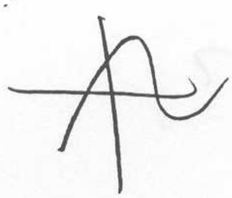
$m = \frac{y_1 - y_1}{x_1 - x_2}$

$0 = \frac{2-y}{0-x}$

$0 = 2 - y$

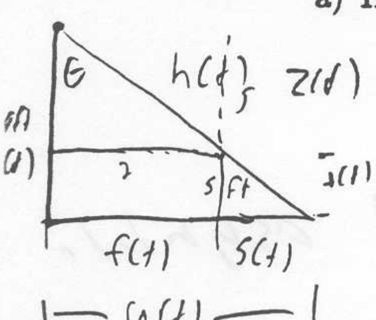
$\boxed{y = 2}$

-2



[4] **3. A street light is mounted at the top of a 10 foot pole. A 5 ft tall woman walks towards the pole with a speed of 3 ft/s along a straight path (until she gets to the pole).**

a) How fast is the tip of her shadow moving when she is 2 ft from the pole?



$f(t) = 2ft$

$\frac{df(t)}{dt} = -3ft/s$ ✓

$h(t)^2 = p(t)^2 + y(t)^2$

"10ft the pole height doesn't change with time

-1.5

what is $p(t)$?

b) How fast is the distance between the tip of her shadow and the top of the street light changing when she is 2 ft from the pole?

$h t = \sqrt{5^2 + 2^2}$

$h(t) = \sqrt{29}$

$h(t)^2 = f(t)^2 + (p(t) - 1(t))^2$

[4] **4. Evaluate the following limits:**

a) $\lim_{x\to 2} \frac{\sin(x-2)}{x^2-2} = \frac{\sin 0}{2} \cancel{X} \frac{1}{2}$

-1

b) $\lim_{x\to\infty} \frac{5x-\sqrt{x}-2x^{5/3}}{3x^{5/3}+7x^{1/2}} = \frac{\frac{5x}{x^{5/3}} - \frac{x^{1/2}}{x^{5/3}} - \frac{2x^{5/3}}{x^{5/3}}}{\frac{3x^{5/3}}{x^{5/3}} + \frac{7x^{1/2}}{x^{5/3}}} = -\frac{2}{3}$ ✓

$x = e^{\ln x}$

[4] **5. Find all vertical and horizontal asymptotes (show all your work) for:**

a) $f(x) = 5 - 2e^{-x}$

$\lim_{x\to-\infty} 5-2e^{-x} = -\infty$ ✓

has a horizontal asymptote at $y=5$.

$\lim_{x\to\infty} 5-2e^{-x} = 5$ ✓

why no vert asymptotes?

-1/2

b) $f(x) = \ln(x^2-1)$

$\lim_{x\to\infty} \ln(x^2-1) = 0$ X

has a vertical asymptote at 0 X

and horizontal at 0.

$\lim_{x\to-\infty} \ln(x^2-1) = \infty$ ✓

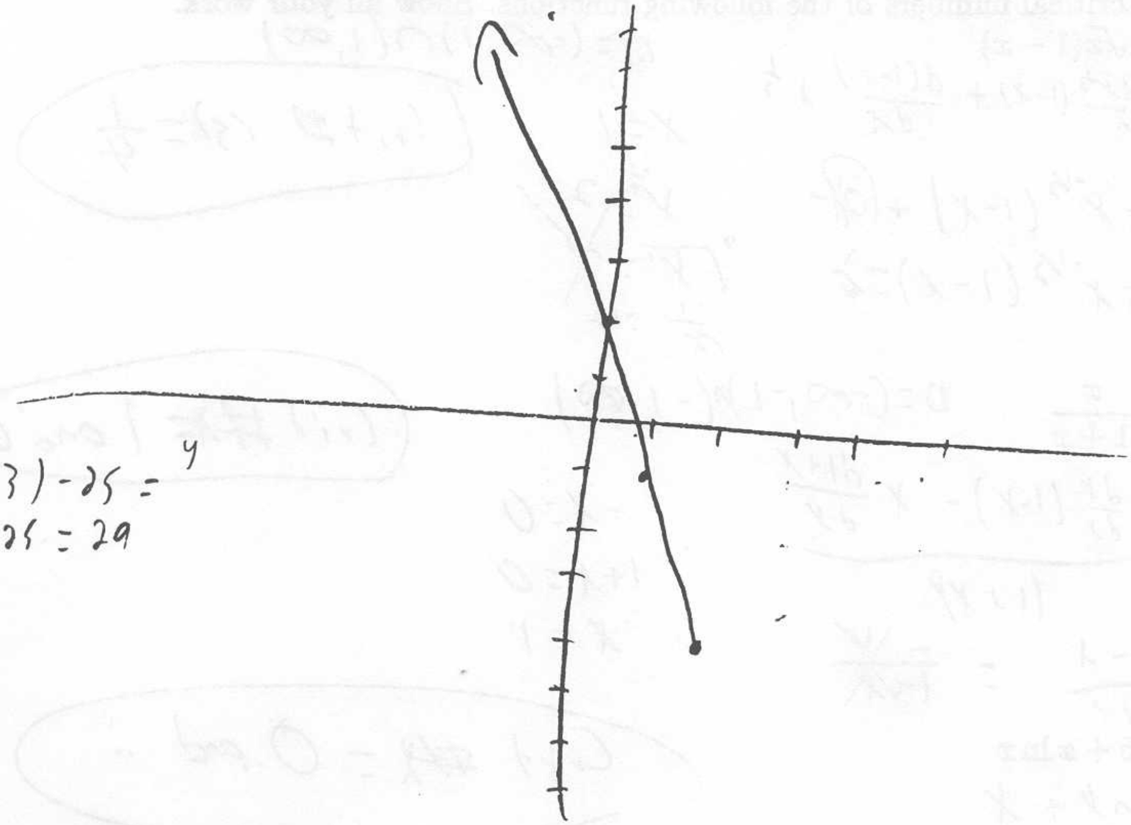
$\lim \ln(x^2-1) = 0$

1

[2] **6. For what numbers is the following function differentiable? Justify your answer.**

(3,29)

$$g(x) = \begin{cases} -3x+2 \ , & x \le 2 \\ 3x^2+2 \ , & 2 < x \le 3 \\ 18x-25 \ , & x > 3 \end{cases}$$ (2,14)



$18(3)-25 =$
$54-25 = 29$

This function g(x) is differentible every were exept for $x=2$ this is because the graph is not Continuous and can't be differentiated

$(-\infty, 2) \cup (2, \infty)$ why?